23
29
20
27
19

25

19) $a_n = ne^{-n}$

$$\lim_{n\to\infty} \frac{n}{e^n} \overset{LH}{=} \lim_{n\to\infty} \frac{\frac{d}{dn}(n)}{\frac{d}{dn}(e^n)}$$

$$= \lim_{n\to\infty} \frac{1}{e^n} = 0$$

$\therefore$ converges to 0

L'Hospital's Rule

alt spelling: l'Hôpital's

Consider $\lim\limits_{\substack{x \to \infty \\ (-\infty) \\ (a)}} \frac{P(x)}{Q(x)}$.

If $\lim\limits_{x \to ?} \frac{P(x)}{Q(x)}$ is "of the indeterminate form $\frac{\infty}{\infty}$"

then $\lim\limits_{x \to ?} \frac{P(x)}{Q(x)} = \lim\limits_{x \to \infty} \frac{P'(x)}{Q'(x)}$

$y = \frac{2x}{3x}$

$\lim\limits_{x \to \infty} \frac{2x}{3x}$ " = " $\frac{\infty \text{ish}}{\infty \text{ish}}$

$= \lim\limits_{x \to \infty} \frac{2}{3} = \frac{2}{3}$

indeterminate forms

$\frac{\infty}{\infty}$ L'H ✓

$\frac{0}{0}$ L'H ✓

23) $\lim_{n\to\infty} \frac{n2^n}{3^n} \overset{LH}{=} \lim_{n\to\infty} \frac{\frac{d}{dn}(n2^n)}{\frac{d}{dn}3^n}$

$\frac{d}{dx}(b^x) = (\ln b)b^x$

$2^n = e^{\ln 2^n} \Rightarrow e^{n\ln 2}$

$= \lim_{n\to\infty} \frac{2^n + (n\ln 2)2^n}{}$

yuck

$\frac{d}{dx}(e^{n\ln 2}) = (e^{n\ln 2})\cdot\left(\frac{d}{dn}(n\ln 2)\right)$

$= (e^{n\ln 2})(\ln 2)$

$= (\ln 2)2^n$

$\lim_{n\to\infty} \frac{n}{\left(\frac{3^n}{2^n}\right)}$

$\frac{n}{1}\cdot\frac{2^n}{3^n}$

$= \lim_{n\to\infty} \frac{n}{\left(\frac{3}{2}\right)^n}$

$\lim_{n\to\infty} \frac{1}{\ln\left(\frac{3}{2}\right)\cdot\left(\frac{3}{2}\right)^n}$

$= 0$

aka L'Hôpital's

L'Hospital's Rule

If $\lim_{x\to\infty} \frac{P(x)}{Q(x)}$ is "of the indeterminate form $\frac{\infty}{\infty}$" then
($-\infty$, a)

$$\lim_{x\to\infty} \frac{P(x)}{Q(x)} = \lim_{x\to\infty} \frac{P'(x)}{Q'(x)}$$

$$\lim_{x\to\infty} \frac{2x}{3x}$$

$$= \lim_{x\to\infty} \frac{2}{3} = \frac{2}{3}$$

Indeterminate Forms

$\frac{\infty}{\infty}$ L'H ✓

$\frac{0}{0}$ L'H ✓

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19) $a_n = ne^{-n} = \frac{n}{e^n}$

$$\lim_{n\to\infty} \frac{n}{e^n} = \lim_{n\to\infty} \frac{\frac{d}{dn}(n)}{\frac{d}{dn}(e^n)}$$

$$\overset{LH}{=} \lim_{n\to\infty} \frac{1}{e^n} = 0$$

consider $\frac{n}{e^n}$

25) $\lim_{n\to\infty} n \sin\frac{1}{n}$

$$\lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}}$$

$$\overset{LH}{=} \lim_{n\to\infty} \frac{(\cos\frac{1}{n})(\frac{-1}{n^2})}{(-\frac{1}{n^2})}$$

$$= \lim_{n\to\infty} \cos\frac{1}{n}$$

$$= 1$$

$$\lim_{n\to\infty} \frac{\sin(\frac{1}{n})}{\frac{1}{n}}$$

$u = \frac{1}{n}$

as $n\to\infty$

$u\to 0$

$$\lim_{u\to 0} \frac{\sin u}{u} = 1$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

8.1 25)

$$\underbrace{\lim_{n\to\infty} n \sin\frac{1}{n}}$$

L'H Rule requires

$$\frac{P(x)}{Q(x)}; \quad \frac{\infty}{\infty} \text{ or } \frac{0}{0}$$

$$= \lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}} \qquad \frac{d}{dn} n^{-1} = -1(n^{-2})$$

$$\overset{LH}{=} \lim_{n\to\infty} \frac{\cos\left(\frac{1}{n}\right)\cancel{\left(\frac{-1}{n^2}\right)}}{\cancel{\left(\frac{-1}{n^2}\right)}}$$

$$= \lim_{n\to\infty} \cos\left(\frac{1}{n}\right) = 1$$

$$\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\left(\frac{1}{n}\right)}$$

$u = \frac{1}{n}$ ✓

as $n \mapsto \infty$

$u \mapsto 0$ ✓

$$\boxed{\lim_{u\to 0} \frac{\sin u}{u}} = 1$$

8.1/39 $a_n = \frac{3n^2-2}{n^2+1}$ show its bounded.

for $n \geq 1$

$\frac{3n^2-2}{n^2+1}$ is positive.

consider $n \geq 1$

$$-2 \leq \frac{3n^2-2}{n^2+1} < \frac{3n^2}{n^2+1} < \frac{3n^2}{n^2} = 3$$

$$\lim_{n\to\infty} \frac{3n^2-2}{n^2+1} = L$$

$$\lim_{n\to\infty} \frac{3n^2-2}{n^2+1} = 3$$

8.1/41)

$$a_n = \frac{\sin(n^2)}{n+1}$$

consider $n \geq 0$

$$-1 \leq \frac{\sin(n^2)}{n+1} \leq \frac{1}{n+1} \leq 1$$

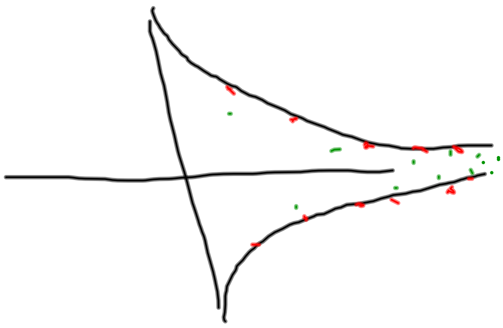
seq 33 $a_n = (-1)^n \frac{e^{-n}}{n}$

☆ $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{e^{-n}}{n} = \lim_{n\to\infty} \frac{1}{ne^n} = 0$

by ~~~ $\lim_{n\to\infty} a_n = 0$

$\frac{e^{-n}}{n} \leq \frac{1}{n}$

$-\frac{1}{n} \leq (-1)^? \frac{e^{-n}}{n} \leq \frac{1}{n}$



$\lim_{n\to\infty} \frac{1}{n} = 0$

43)

a_1	a_2	a_3	a_4	a_5
$\frac{1}{8}$	$-\frac{1}{4}$	$\frac{1}{2}$	-1	2

2^{-3} $\quad -2^{-2}$ $\quad 2^{-1}$ $\quad -2^{0}$ $\quad 2^{1}$ $\quad \ldots$

$2^{1}\cdot\frac{1}{2^4}$ $\quad 2^{2}\cdot\frac{1}{2^4}$ $\quad \ldots$

rule: $a_n = (-1)^{n+1}\, 2^{(n-4)}$

$-2^{-3}\cdot$ $\quad 1=2^{0}$ $\quad -2=2^{1}$ $\quad 4=2^{2}$ $\quad -8=2^{3}$ $\quad 16$

$1 \quad 2 \quad 3 \quad 4$

2^{n-1}

35) inc or dec? $\frac{n+3}{n+2}$?

$n^{th} > (n+1)^{st} \longrightarrow$ dec

$$\frac{n+3}{n+2} > \frac{(n+1)+3}{(n+1)+2}$$

for $n \geq 0$

$$\left(\frac{n+3}{n+4}\right)\frac{n+3}{n+2} \overset{?}{>} \frac{n+4}{n+3}\left(\frac{n+3}{n+4}\right)$$

$$\frac{(n+3)^2}{(n+2)(n+4)} \overset{?}{>} 1$$

$$\frac{n^2+6n+9}{n^2+6n+8} \overset{?}{>} 1$$

$$\left(\frac{n^2+6n+8}{n^2+6n+9} = \frac{n^2+6n+9}{n^2+6n+9} - \frac{1}{n^2+6n+9}\right.$$

$$1 - \frac{1}{n^2+6n+9}$$

$f(x)$: Consider $\frac{x+3}{x+2} = \frac{x+2+1}{x+2} = 1 + \frac{1}{x+2}$

$$f'(x) = \frac{-1}{(x+2)^2} < 0$$

$f'(x) < 0 \Rightarrow f(x)$ decreasing

37) $a_n = \frac{e^n}{n}$

$\frac{e^n}{n} \; ? \; \frac{e^{n+1}}{n+1}$

$\frac{e^{n+1}}{n+1} < \frac{e^{n+1}}{n}$

$> \frac{e^{n+1}}{n} \cdot \frac{1}{e}$

$= \frac{e^n}{n}$

problem

Consider $\frac{e^x}{x}$

$f'(x) = \frac{xe^x - e^x}{x^2} = \frac{(x-1)e^x}{x^2}$

$- \; - \; - \;|\; - \; \cdots \;|\; + \; + \; + \; +$

$0 \qquad 1$

$x > 1$

$f'(x) > 0$

$\therefore$ inc.

$\therefore$ $f(x)$ increases $[1, \infty)$.

8.1
38)

$$\left\{\frac{3^n}{(n+2)!}\right\}_{n=1}^{\infty}$$

$$\frac{3}{6}, \frac{9}{24}, \frac{27}{120}, \frac{81}{720},$$

$n!$

$a_{n+1} \geq a_n$

$a_{n+1} - a_n \geq 0$

$\frac{a_{n+1}}{a_n} \geq 1$

$3\cdot 2\cdot 1$

$\Gamma(x)$

$\Gamma(n) = n!$

$a_{n+1} \qquad a_n$

$$\frac{3^{(n+1)}}{[(n+1)+2]!} \;?\; \frac{3^n}{(n+2)!}$$

$$\frac{3\cdot 3^n}{(n+3)(n+2)!} \;?\; \frac{3^n}{(n+2)!}$$

$$\frac{3}{n+3}\cdot a_n < a_n$$

$4! = 4\cdot 3!$

$4(3\cdot 2\cdot 1)$

$(n+1)! = (n+1)\,n!$

$(n+1)(n)(n-1)(n-2)\cdots(1)$

$\frac{3}{n+3} < 1$ when $n \geq 0$